Table 1

R_T	ρ_0	C_1	C_2	$Ra\ Ga_c$	Ra
1.5	$\left[\frac{1.25}{1.5-0.5y}\right]^{-1}$	−0.4	1.2	101.68	40.672
2	1	0	1	58.9267	39.284
2.5	$\left[\frac{1.6751}{2.5-1.5y}\right]^{1/3}$	0.3031	0.8516	44.32	37.9886
3	$\left[\frac{1.86605}{3-2y}\right]^{1/2}$	0.5477	0.7340	37.2	37.2

$R_T = \frac{T_p}{T_{amb}}$; $T_{amb} = 300\,K$; $S = 3.15$.

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ANALYSIS OF BATCH SCRAPED SURFACE HEAT EXCHANGE

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NOMENCLATURE

- A, imperfect mixing parameter: ratio of that fraction of the scraped layer which is well mixed with the core: $vm/(vm + M)$;
- B', perfect scraping parameter: $\frac{4}{R}\frac{\sqrt{\alpha\theta_c}}{\pi}$;
- B, parameter including external heat transfer resistance $\frac{U}{h_f}\frac{4}{R}\frac{\sqrt{\alpha\theta_c}}{\pi}$;
- C_p, heat capacity [J/gK];
- h_e, heat transfer coefficient between external fluid (Bath) and wall;
- h_f, theoretical scraped surface heat transfer coefficient: $2k/\sqrt{\pi\alpha\theta_c}$ [W/m^{2}K];
- k, thermal conductivity of mixture of fluid [W/mK];
- k_w, thermal conductivity of heat exchange wall 16.5 [W/mK];
- l, thickness of heat exchanger wall (1.01 mm);
- m, mass of scraped layer per scraping cycle [g];
- M, mass of central (mixed) core [g];
- n, total number of scrapings (θ/θ_c);
- R, radius of cylindrical scraped surface heat exchanger;
- U, overall heat transfer coefficient between external bath and scraped surface heat exchanger wall:
  $$\frac{1}{U} = \frac{1}{h_e} + \frac{l}{k_w} + \frac{\delta_i}{k} + \frac{1}{h_f}\ [\mathrm{W/m^2K}];$$
- T, average temperature of well mixed core material [°K];
- T_w, wall or bath temperature [K];
- T_0, initial temperature of mixture system [K].

Greek symbols

- α, thermal diffusivity of mixture of fluid [m^2/s];
- δ_i, thickness of residual film left by the scraper blade during imperfect scraping [m];
- δ, scraped film thickness (scraper blade width) [m];
- v, ratio of mass of intermediate mixing layer to scraped layer;
- θ, total time $(n\theta_c)$ [s];
- θ_c, time between scrapings [s].

INTRODUCTION

WE HAVE made an analysis [1,2] of scraped surface heat exchange [3–12] in a cylindrical batch vessel which extends the traditional suggestion of Houlton [13] to include the effects of imperfect scraping and incomplete mixing between the material removed by the scraping and the main body of the heated material. There is current interest in the process [14] because the increase in heat transfer by scraping can be obtained for less mechanical energy than that required by stirring or agitation with baffles when the material has a thermal conductivity less than 1 W/mK, even if it is not very viscous [15].

When the process is regarded as a sequence of contacts with the wall and subsequent perfect mixing with a well mixed core, one obtains the average dimensionless temperature of the layer for the nth scraping as

$$\frac{T_a^{n+1} - T^n}{T_w - T^n} = \frac{2k\sqrt{\theta_c}}{\sqrt{(\pi\alpha)}\rho\delta C_p}. \tag{1}$$

Perfect mixing implies

$$mC_p(T_a^{n+1} - T^n) = (M + m)C_p(T^{n+1} - T^n). \quad (2)$$

For a cylindrical system

$$m = 2\pi R\rho\delta,\ M = \pi(R - \delta)^2\rho.$$

The scraper usually removes a thickness far greater than the penetration of the temperature profile. Under these circumstances, the theory implies that the heat transfer does not depend upon the width of the scraper (2), and

$$\frac{T^{n+1} - T^n}{T_a^{n+1} - T^n} = \frac{m}{m + M} \approx \frac{2\delta}{R} \quad \text{because } \delta \ll R \quad (3)$$

using (1):

$$T^{n+1} - T^n = B(T_w - T^n) \text{ with } B' = \frac{4}{R}\sqrt{\left(\frac{\alpha\theta_c}{\pi}\right)}. \quad (4)$$

This linear difference equation with $T = T_0$ for $n = 0$ has the solution

$$\frac{T^n - T_w}{T_0 - T_w} = (1 - B')^n \approx \exp(-B'n) \quad (5)$$

which implies the same heat transfer coefficient as Houlton [13]. If T_w exists within some external bath, instead of at the scraped wall, the overall heat-transfer coefficient appears in

$$B = \frac{U}{h_f}\frac{4}{R}\sqrt{\left(\frac{\alpha\theta_c}{\pi}\right)}.$$

In imperfect mixing, flow patterns suggest an intermediate mixing layer at the inside edge of the scraper. The scraped layer mixes with this region whose temperature lies between that of the wall and the core. This layer has a mass vm and the scraped layer a mass m, and the mixed temperature is

$$vmC_pT_\delta^n + mC_pT_a^{n+1} = (v + 1)mC_pT_\delta^{n+1}, \quad (6)$$

and

$$T_\delta^{n+1} = (vT_\delta^n + T_a^{n+1})/(v + 1) \quad (7)$$

where T_a is the average temperature of the scraped layer, and T_δ is the temperature of the intermediate layer. The scraped layer is returned to the wall, and the intermediate layer mixed with the main body

$$(vm + M)C_pT^{n+1} = MC_pT^n + vmC_pT_\delta^{n+1}$$

and

$$T^{n+1} = T^n + \frac{vm}{M + vm}(T_\delta^{n+1} - T^n)$$

$$= T^n + A(T_\delta^{n+1} - T^n);\ A = \frac{vm}{M + vm}. \quad (8)$$

The two simultaneous difference equations to be solved are then

$$T_\delta^{n+1} - T_\delta^n = B(T_w - T_\delta^n) \quad (9)$$

$$T^{n+1} - T^n = A(T_\delta^{n+1} - T^n). \quad (10)$$

With $T = T_0$ for $n = 0$, the result is equation (11). In general, the singularity implied by the denominator will not be approached in practical situations

$$\frac{T - T_w}{T_0 - T_w} = \frac{B}{B - A}(1 - A)^{n+1} + \frac{A}{A - B}(1 - B)^{n+1} \quad (11)$$

Because the value for n is large, and the values for the parameters A and B are usually much less than unity, the exponential approximation can be used to obtain the final form

$$\frac{T - T_w}{T_0 - T_w} \approx \frac{B}{B - A}\exp(-An) + \frac{A}{A - B}\exp(-Bn). \quad (12)$$

Experimental results showing the temperature approach as a function of time for different scraping speeds and mixture types provide the test of the theory.

There are other studies of the power consumption as a function of mixture properties which indicate the energy required to accomplish the scraping [16–19], but this study is concerned with the understanding of the heat transfer alone.

EXPERIMENTAL INVESTIGATION

The experimental apparatus was a modified laboratory size grease compounding kettle (0.15 m dia, 0.15 m high) with a two bladed shaft (speeds of 10.5, 30 and 60 rev/min). Temperature changes of 40–90 K were recorded to 0.5 K as a function of time (20 min to 2 h) using a five junction 30 gage copper–constantan thermopile connected to a 10 mV full scale recorder. The details of the apparatus and procedure are in Park [2]. A 5 mm screen attached to the inner edge of the 25 mm wide scrapers with 10 mm baffles reduced the flow back to the wall from behind the scraper, and improved the mixing between the scraped layer and the mixed core.

The fluids used were mixtures of Cabosil (product of Cabot Corporation) silica flakes (1–20% by weight) in toluene. Mixture properties were calculated from those of the pure materials using the method of Hamilton [20]. For the reported mixture (2.3% by weight Cabosil), $\alpha = 0.813\text{E-}7\ m^2/s$, $k = 0.157$ W/mK, and for water $\alpha = 1.6\text{E-}7\ m^2/s$ and $k = 0.6$ W/mK. Park [2] investigated the effect of scraping frequency, blade design, circulation control, imperfect scraping action, external heat transfer resistance, imperfect mixing.

RESULTS AND DISCUSSION

Typical results are shown for the mixture in Fig. 1. Measurements of the metal wall temperature along with those of the external bath permitted the determination of the external heat-transfer coefficient between the bath and the wall. Even stirring the bath mixture of ice and water failed to reduce this heat-transfer coefficient below 1022 W/m^2K. When this value is used for the parameter B' in equation (5) the results are the straight lines in Fig. 1. The values at 60 rev/min agree within 5% but those at lower speeds are not

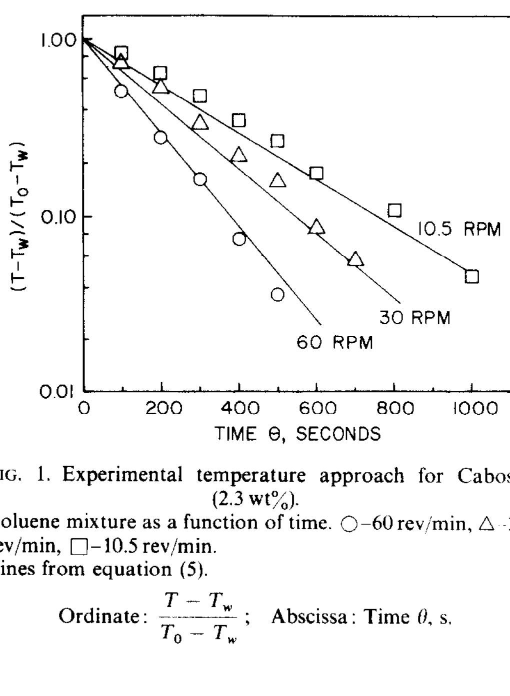

FIG. 1. Experimental temperature approach for Cabosil (2.3 wt%).
Toluene mixture as a function of time. ○–60 rev/min, △–30 rev/min, □–10.5 rev/min.
Lines from equation (5).

Ordinate: $\frac{T - T_w}{T_0 - T_w}$; Abscissa: Time θ, s.

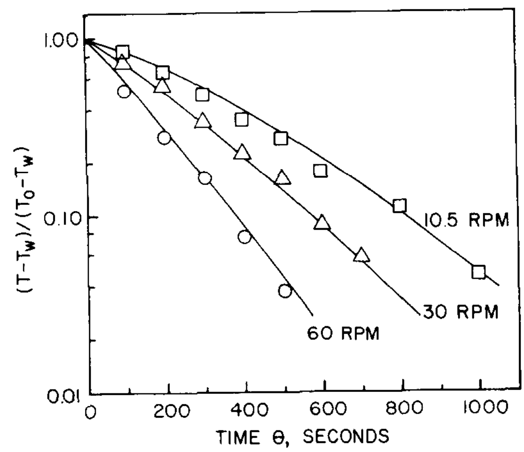

FIG. 2. Experimental temperature approach for Cabosil (2.3 wt%).
Toluene mixture as ∂ function of time. ○–60 rev/min, △–30 rev/min, □–10.5 rev/min.
Lines from equation (4)

Ordinate: $\frac{T - T_w}{T_0 - T_w}$; Abscissa: Time θ, s.

satisfactory. Agreement can be obtained for most of the points assuming imperfect scraping (0.017 mm residual film at 60 rev/min, 0.114 mm at 30 rev/min and 0.068 mm at 10.5 rev/min). However the lowest points, representing the longest contact times and closest approach to the external temperature, still lie below the theory. Although the imperfect scraping idea requires only very thin residual films, this interpretation was not accepted because no Cabosil could be seen attached to the wall immediately behind the scraper as it passed, and because the closest approach points lay below the theoretical line of equation (5). Presuming perfect scraping [3–12] and using the value of 0.023 for A in equation (12) gives the lines of Fig. 2. The lowest points correspond to about 98% approach. These results indicate that the effect of external heat transfer resistance, scraping frequency, imperfect mixing can be properly accounted for using equation (12) and the properties of the mixture and apparatus.

While Trommelen *et al.* [21] have suggested that for flow apparatus it is appropriate to consider a boundary layer built on the scraper blade, we find for batch systems that accounting for imperfect mixing is sufficient. We also suggest that clean scraping with good control of the mixing in the intermediate layer and the core will accomplish the heat transfer with less energy consumption.

CONCLUSION

For batch cylindrical scraped surface heat exchangers with viscous material, temperature change with time can be predicted by equation (12) provided the effects of external heat transfer resistance, exchanger wall resistance, imperfect mixing are included.

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